AMENDMENTS TO THE SPECIFICATION

Please amend the paragraph beginning at line 8 on page 2 as follows:

Maxwell (1858) founded modern kinetic theory when he introduced his new concept of the molecule velocity distribution function and deduced its equilibrium form. Maxwell (1867) derived the Maxwell transport equations and showed that the collision change integrals could be solved analytically without knowing the molecule velocity distribution function for Maxwell molecules. Boltzmann (1872) derived his integro-differential equation for Maxwell's molecular velocity distribution function and solved for his famous H-theorem, which proved that Maxwell's equilibrium form was correct. Chapman (1916), using Maxwell's transport equations, determined accurate general formulae for the gas transport coefficients. Enskog (1917) gave a general solution method for he the Boltzmann equation, which showed that the Euler equations were the first and the Navier Stokes equations were the second approximate solutions to the Boltzmann equation and gave gas transport coefficients identical to Chapman's. Both the Chapman and Enskog analyses were for isotropic perfect gases, being based on perturbations of Maxwell's isotropic equilibrium solution. Their independent analyses cemented the belief in both the scientific and particularly the engineering professions that the Navier Stokes equations were generally and universally valid and that the correct way of analyzing both laminar and turbulent flows was through the Navier Stokes equations. This belief has been engraved in stone in both the scientific and engineering literature throughout the 20th century.

Please amend the paragraph beginning at line 1 on page 7 as follows:

This disclosure concerns a simple method of deriving the correct kinetic anisotropic fluid turbulent flow equations from the Boltzmann equation and the correct thermal and turbulent

moment closures to yield a new, closed set of fluid dynamic equations describing anisotropic

turbulent motions, solving the ~~century~~ age old quest for the solution to this classical problem.
Please amend the paragraph beginning at line 8 on page 10 as follows:

The Boltzmann equation may be written in fluid dynamic collision rate form: [See

~~Appendix,~~ Equation set 1];

where $\dfrac{D}{Dt} = [\rho F(v)]$ is the Lagrangian convective motion derivative, ρ is the fluid density, υ is the

molecular collision rate, F(v) is the molecular velocity (v) distribution function and F(v)* is the

molecular velocity distribution function collided into F(v). There is no approximation in writing the

Boltzmann collision integral in this format, since the required thermal velocity moments of F(v)*

are correctly calculated from the full Boltzmann collision integral (Kliegel (1990)). The molecular

collision rate is

$$\upsilon = \rho K$$

for Maxwell molecules, where K is a molecular collision constant.

Please amend the paragraph beginning at line 6 on page 11 as follows:

Multiplying the Boltzmann equation by the molecular velocity product $v_1^l v_2^m v_3^n$, where *l, m,*

n are integers, and integrating over all possible velocities (indicated by the bar) yields the Maxwell

molecular velocity moment equation: [See ~~Appendix,~~ Equation Set 2]. Derivation of the

anisotropic turbulent flow equations from this equation will be discussed later.

Please amend the paragraph beginning at line 10 on page 11 as follows:

Consider the molecular velocity (v) to be composed of a mass mean fluid flow velocity (u)

and a thermal motion velocity (c), then the Cartesian components of these velocities are related by

$$v_1 = u_1 + c_1$$

Multiplying the Boltzmann equation by the thermal velocity product $c_1^l c_2^m c_3^n$, where l, m and n are integers, and integrating over all possible thermal velocities (indicated by the bar) yields the Maxwell thermal velocity moment equation: [See ~~Appendix,~~ Equation Set 3].

Please amend the paragraph beginning at line 16 on page 11 as follows:

For odd moment $(l + m + n = 2N + 1, \; N \text{ integer})$ equations, the coefficients of the density gradient terms are even moments and products of lower even moments, which have equilibrium values. Thus these even moment coefficients must be zero to Burnett order for the density gradient terms to be of Burnett order. This requirement yields the general even order (fourth, sixth and higher) density gradient independent thermal moment closure relationships: [See ~~Appendix,~~ Equation Set 4].

Please amend the paragraph beginning at line 22 on page 11 as follows:

The fourth order thermal moment closure relationships can also obtained from Kliegel (1990) by simple calculation but he did not state these results nor utilize them in any manner. Substituting these relationships into Equation Set 3, one obtains the <u>closed</u> odd order (third, fifth and higher) density gradient independent thermal moment equation. [See ~~Appendix,~~ Equation Set 5].

Please amend the paragraph beginning at line 4 on page 12 as follows:

The classical Maxwell thermal velocity twenty moment equation set is: [See ~~Appendix,~~ Equation Set 6].

Please amend the paragraph beginning at line 6 on page 12 as follows:

After using the fourth order moment density gradient independent closure relationships: [See ~~Appendix,~~ Equation Set 7], the closed directional thermal energy flux and structure equations become: [See ~~Appendix,~~ Equation Set 8] where the fourth order thermal moments have been replaced by products of second order moments through the fourth order density gradient independent closure relationships.

Please amend the paragraph beginning at line 21 on page 12 as follows:

The twenty moment equations may be further simplified by performing an Enskog-type ordering analysis on all terms to obtain the reduced equation set: [See ~~Appendix,~~ Equation Set 9]

Please amend the paragraph beginning at line 11 on page 14 as follows:

The instantaneous Maxwell molecular velocity (v) and the thermal velocity (c) moment equations (Equation Sets 2 and 3) are both valid at every instant of time and the instantaneous Maxwell thermal velocity moment equations can be derived from the instantaneous Maxwell molecular velocity moment equations. The time average Maxwell molecular velocity (v) and time average thermal velocity (c) moment equations are also both valid simultaneously. The time average thermal velocity equations yield equations governing the behavior of the anisotropic time average thermal directional kinetic energies, shears, directional kinetic energy fluxes and structure correlations: [See ~~Appendix,~~ Equation Set 10].

Please amend the paragraph beginning at line 3 on page 15 as follows:

The time average molecular velocity equations yield equations governing mass and momentum conservation and the behavior of both the time average thermal and turbulent directional energies, shears, energy fluxes and structure correlations: [See ~~Appendix,~~ Equations Set 11].

Please amend the paragraph beginning at line 7 on page 15 as follows:

After eliminating the collision change terms with the time average thermal Maxwell moment equations and other simplifications, one obtains the equations governing the behavior of the anisotropic time average turbulent directional kinetic energies, shears, directional energy fluxes and structure correlations: [See ~~Appendix,~~ Equation Set 12].

Please amend the paragraph beginning at line 11 on page 15 as follows:

Both sets of third order directional kinetic energy fluxes and structure correlation equations are closed using the fourth order density gradient independent time average thermal and turbulent moment closure relationships: [See ~~Appendix,~~ Equation Sets 13 and 14].

Please amend the paragraph beginning at line 14 on page 15 as follows:

One thus obtains the complete closed Maxwell twenty moment equation set governing anisotropic time average turbulent flows: [See ~~Appendix,~~ Equation Set 15].

Please amend the paragraph beginning at line 16 on page 15 as follows:

By the same process, corresponding higher order time average turbulent moment equation sets may be derived using higher order closure relationships. The general density gradient

independent time average thermal and turbulent even moment closure relationships are: [See ~~Appendix,~~ Equation Sets 16 and 17].

Please amend the paragraph beginning at line 3 on page 16 as follows:

The above results may be simply extended in many ways. For example, the general even order density gradient independent moment closure relationships (Equation Set 4) are valid in equilibrium. Thus, [See ~~Appendix,~~ Equation Set 18] and the even order collision integrals $\left(\overline{c_1^{l+1} c_2^m c_3^n} - \overline{c_1^{l+1} c_2^m c_3^n}, \text{etc.} \right)$ may be calculated for all molecular models.

Please amend the paragraph beginning at line 1 on page 17 as follows:

While all equation sets may be solved by straightforward computation, they may be more efficiently solved using the method described in my co-pending application entitled "Method for Algebraically Solving Differential Equations, Including Stiff Equations, to High Accuracy" <u>Serial No. 09/654,004 filed September 1, 2000</u>.

Please amend the top of page 18 as follows:

~~APPENDIX~~